

# SO(10): a theory of fermion masses and mixings

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## Abstract

SO(10) grand unified theory seems to have all the ingredients to be a complete unified theory of quarks and leptons. I review here its minimal, possibly realistic versions, both supersymmetric and not.

## 1 Introduction

This talk I delivered in July last year in honor of Gustavo Branco coming of age. Gustavo and I overlapped in graduate school at City College of New York where we shared the same advisor Rabi Mohapatra who also came of age last year. City College in the seventies was a great place, unique in a sense: you could do good physics and at the same time teach local kids from Harlem. And of course in the seventies physics was really exciting, standard model, asymptotic freedom and all that, and one could think about L-R symmetry, neutrino mass, strong CP and many other interesting issues.

## 2 SO(10): some generic features

Grand unification, a theory of strong and electroweak interactions based on the single gauge group, implies two remarkable physical phenomena: proton decay and magnetic monopoles. More than 30 years later neither has been found, a fact that seems to render the search of the theory impossible. Fortunately, the accumulated information on fermion masses and mixings may provide the necessary clue. The minimal GUT group SU(5) [2] fails to unify the family of matter (the fermions) and in its minimal form predicts massless neutrinos. On the other hand, the minimal theory of matter unification, SO(10) [3], predicts massive neutrinos and through the seesaw mechanism [4] explains why neutrinos are so light. While SU(5) can be made to work with enough gymnastics, it by itself cannot connect quark and lepton mixings (for a recent study of ordinary SU(5) see e.g. [5]). The minimal predictive theory must be based on SO(10).

Minimal or not, the  $SO(10)$  theory is certainly appealing on theoretical grounds. Besides unifying the family of fermions, and explaining the smallness of neutrino mass, it has left-right symmetry [6] as a finite gauge transformations (broken spontaneously) in the form of generalized charge conjugation and a Pati-Salam  $SU(4)_c$  symmetry which unifies quarks and leptons. The supersymmetric version has R-parity (matter parity) as a gauge symmetry [7], a part of the center  $Z_4$  of  $SO(10)$ . In some case (tree level see-saw) it can be shown [8] that R-parity remains exact at all energies, surviving thus all the symmetry breaking. The lightest supersymmetric particle (LSP) is then stable, a perfect dark matter candidate.

The fact that  $SO(10)$  by itself may naturally account for all fermion masses and connect small quarks and large lepton mixings is a strong argument in its favor. Here I offer a short updated review of the great effort in this direction. For more information and references see [9, 10].

### 3 $SO(10)$ and only $SO(10)$ : can it be a complete theory (forgetting gravity)?

The fermion families are 16-dimensional spinors of  $SO(10)$ , and from

$$16_F \times 16_F = 10_H + 120_H + 126_H \quad (1)$$

one has three possible Yukawa coupling matrices, enough to fit all the fermion masses and mixings. Now, if one wants a predictive theory, one should stick to the minimal one. Better to say, one could use the information on masses and mixings to determine the theory, and thus make up for the absence of proton decay and monopole information. I want to describe and advocate this program here.

The point is simple. Ideally one could try a single Higgs, but a single set of Yukawas can be diagonalized and then all the fermion mass matrices would be simultaneously diagonal. Beside bad mass relations, this would imply no quark and no lepton mixings in the weak currents. The minimal theory must thus have two such Higgses (at least); in other words two Yukawa matrices. Compared to at least four in the standard model, it should be no surprise that such a theory is over constrained and predictive. In the next section I go through three such possible theories, one of which was studied at great length. The most interesting feature of these predictive theories is that they

seem to determine the low energy (TeV) effective theory, i.e. decide whether or not one has a low energy supersymmetry, split supersymmetry [11] or no supersymmetry at all. No need for philosophical arguments in favor or against supersymmetry.

This ambitious program of making do without any new physics beyond the GUT itself is often praised. It is also often criticized for the same reason, namely for ignoring all the higher dimensional operators which would emerge from the physics above  $M_{GUT}$ . After all, such effects are in principle of the order  $M_{GUT}/M_{Pl} \approx 10^{-3}$ , and thus potentially large compared to the first, and comparable to the second generation Yukawa couplings. However, such effects could be further suppressed by small dimensionless couplings. A nice example is the  $d = 5$  proton decay

$$O_{\Delta B \neq 0} = c Q Q Q L / M_{Pl} , \quad (2)$$

which follows from the SO(10) invariant interaction

$$O_S = c 16_F^4 / M_{Pl} . \quad (3)$$

The proton longevity  $\tau_p > 10^{33}$  yr implies  $c < 10^{-6}$ . Unless we play a texture game, with such small coefficients all other physical effects can be safely ignored.

## 4 The models

We need to decide here which combination of  $10_H$ ,  $120_H$  and  $\overline{126}_H$  is to be chosen (must be two out of three). We have the following possibilities:

- (i)  $\overline{126}_H + 10_H$ ;
- (ii)  $120_H + 10_H$ ;
- (iii)  $\overline{126}_H + 120_H$ ;
- (iv)  $10_H + 10_H$ ;
- (v)  $120_H + 120_H$ ;
- (vi)  $\overline{126}_H + \overline{126}_H$ ;

We will need the Pati-Salam  $SU(2)_L \times SU(2)_R \times SU(4)_C$  decomposition

$$\begin{aligned} 10_H &= (2, 2, 1) + (1, 1, 6) , \\ \overline{126}_H &= (2, 2, 15) + (1, 3, 10) + (3, 1, \overline{10}) + (1, 1, 6) , \\ 120_H &= (2, 2, 15) + (2, 2, 1) + (1, 1, 10) + (1, 1, \overline{10}) + (1, 3, 6) + (3, 1, 6) , \end{aligned} \quad (4)$$

and the fact

$$Y_{10} = Y_{10}^T, Y_{126} = Y_{126}^T, Y_{120} = -Y_{120}^T. \quad (5)$$

With all three Higgs fields  $10_H$ ,  $120_H$  and  $\overline{126}_H$ , one finds

$$\begin{aligned} M_u &= \langle 2, 2, 1 \rangle_{10}^u Y_{10} + \langle 2, 2, 15 \rangle_{126}^u Y_{126} + (\langle 2, 2, 1 \rangle_{120}^u + \langle 2, 2, 15 \rangle_{120}^u) Y_{120}, \\ M_d &= \langle 2, 2, 1 \rangle_{10}^d Y_{10} + \langle 2, 2, 15 \rangle_{126}^d Y_{126} + (\langle 2, 2, 1 \rangle_{120}^d + \langle 2, 2, 15 \rangle_{120}^d) Y_{120}, \\ M_l &= \langle 2, 2, 1 \rangle_{10}^d Y_{10} - 3 \langle 2, 2, 15 \rangle_{126}^d Y_{126} + (\langle 2, 2, 1 \rangle_{120}^d - 3 \langle 2, 2, 15 \rangle_{120}^d) Y_{120}, \\ M_D &= \langle 2, 2, 1 \rangle_{10}^u Y_{10} - 3 \langle 2, 2, 15 \rangle_{126}^u Y_{126} + (\langle 2, 2, 1 \rangle_{120}^u - 3 \langle 2, 2, 15 \rangle_{120}^u) Y_{120}, \\ M_{\nu_R} &= \langle 1, 3, 10 \rangle Y_{126}, \\ M_{\nu_L}^{II} &= \langle 3, 1, \overline{10} \rangle Y_{126}, \end{aligned} \quad (6)$$

where  $M_u$ ,  $M_d$ ,  $M_l$ ,  $M_D$ ,  $M_{\nu_R}$ ,  $M_{\nu_L}$  denote up quark, down quark, charged leptons, neutrino Dirac, right-handed neutrino and left-handed neutrino mass matrices respectively. The left-handed neutrino mass matrix  $M_{\nu_L}^{II}$  is commonly called type II see-saw matrix [12, 13], because the induced vev for  $\langle 3, 1, \overline{10} \rangle$  is small:  $\langle 3, 1, \overline{10} \rangle \approx M_W^2/M_{GUT}$ . Certain obvious features can be read-off from (6):

a) 10 treats quarks and leptons on the same footing, since  $(2,2,1)$  is a  $SU(4)_C$  singlet. This is ideal for the third generation in the case of low energy supersymmetry.

b)  $\overline{126}$  gives us the right-handed neutrino mass and the type I see-saw (and also type II); and furthermore the Georgi-Jarlskog factor [14]  $m_l = -3m_d$  since  $(2,2,15)$  is an adjoint of  $SU(4)_C$ . This works well for the second generation.

c) In the absence of  $\overline{126}$ , neutrinos would only have a Dirac mass, and related to the charged fermion masses. This is cured through the introduction of  $16_H$  needed to break  $B - L$  anyway, since  $16_H \times 16_H = 126_H$  can simulate the direct presence of  $\overline{126}_H$ .

It is easy to see that (iv) predicts  $m_d = m_e$  and (vi)  $3m_d = -m_l$  for all three generations. This is not correct. On the other hand the antisymmetry of  $Y_{120}$  implies  $m_1 = 0$  (the first generation mass) and  $m_2 = -m_3$ , for the case (v). This is clearly wrong and so we are left only with (i)-(iii). Let us summarize next the possibilities, the exhaustive set of candidates for the minimal, realistic  $SO(10)$  GUT.

## 4.1 $\overline{126}_H + 10_H$

This model is emerging as the minimal supersymmetric SO(10) theory [15]. Although it has been around for more than two decades [16, 17], it is only in the last three years that received the proper attention and there is now a dedicated effort in working out the predictions of the theory. It was noticed at the outset [12, 17] that this minimal Higgs Yukawa structure could suffice. However it was only a decade later that it emerged [18] that the theory can provide a deep connection between quark and lepton mixing angles, and the information about the neutrino mass spectrum.

This program was boosted with the observation on a connection between a large atmospheric neutrino mixing angle and  $b - \tau$  unification [19, 20, 21] in the case of type II see-saw. This is easily seen from (6)[22]

$$M_d - M_l \propto Y_{126} \propto M_{\nu_L}^{II} . \quad (7)$$

Let's illustrate what happens for the case of  $2^{nd} - 3^{rd}$  generations.

In the basis for diagonal charged leptons, and for the small down quark mixings (see [19]),

$$M_\nu \propto \begin{pmatrix} m_\mu - m_s & \epsilon \\ \epsilon & m_\tau - m_b \end{pmatrix} . \quad (8)$$

Clearly, a large atmospheric angle requires  $m_b \approx m_\tau$ . This illustrates nicely how a spontaneously broken quark-lepton asymmetry naturally accounts for the small quark and large lepton mixing angles. This is often claimed in the literature to be a mystery for no good reason at all since in the SM there is no connection between quark and lepton properties and certainly no reason to have the same or similar mixings. After all, the neutrino mass ratios are completely different from the charged fermion ones (much less hierarchical); if the mixings are related to mass ratios as hoped, it is more natural to have the mixings rather different.

In any case, we see here that the unified theory of quarks and leptons does precisely that: connects large lepton with small quark mixings. In the following discussion we will offer two more examples which account for the same phenomena, but with different predictions for the masses.

The simplified two-generation analysis above must be performed numerically for the case of three generations. Both type I and type II scenarios seem to work generically and what emerges is the prediction of hierarchical

neutrino masses and an appreciable leptonic 13 mixing angle:  $|U_{e3}| \geq 0.1$  [23, 24, 25]. We have an example here of a predictive theory, good enough to be ruled out. It is remarkable that such a constrained Yukawa sector can account correctly for all the fermion masses and mixings.

However, if one restricts himself to the case of the minimal GUT Higgs sector too, type II seems to run into trouble [26, 27, 28, 29, 30] and possibly type I too [30]. The problem is the compatibility of mass and mixings fittings with unification constraints, a study facilitated by the detailed computation of the full particle spectrum and couplings [31, 32, 33]. This was confirmed recently for the general case, with the same tension between the fermion mass fits and the gauge coupling unification or/and proton decay [34]. Thus this particular version of the minimal SO(10), often coined the minimal supersymmetric SO(10) theory, seems to be in trouble.

However, all this is done by ignoring the effects of soft supersymmetry breaking. Clearly, such terms have a non negligible impact on the first and possibly even the second generation fermion masses. Until they are included, one cannot proclaim the above theory wrong; the trouble is that for generic soft terms the theory is not predictive any more. One could attempt a simpler possibility of flavor blind soft terms at high scale as in gauge mediation models, but even more interesting is to take the theory seriously even as a source of supersymmetry breaking and its transmission to the light sector, along the lines [35]. This rather involved project is being planned.

## 4.2 $\overline{126}_H + 120_H$

Another interesting possibility. It is in contradiction with low energy supersymmetry, since it leads to a prediction  $m_\tau \approx -3m_b$  at  $M_{GUT}$ , far from the  $m_b \approx m_\tau$  of the MSSM. It could in principle work in the ordinary, non-supersymmetric SO(10), where  $2|m_b| \approx |m_\tau|$  at  $M_{GUT}$ . Namely, there are  $m_s/m_b$  ( $m_\mu/m_\tau$ ) effects that should be included, and also in ordinary SO(10),  $M_R$ , the SU(2)<sub>R</sub> breaking scale is around  $10^{13}$  GeV or so, and thus the effects of right-handed neutrinos between  $M_R$  and  $M_{GUT}$  may be appreciable [36]. For this reason we have recently studied this model in the context of the ordinary SO(10) [37].

Again, it is rather useful to get an analytical insight by focusing on the 2<sup>nd</sup> and 3<sup>rd</sup> generations only. Our findings are the following.

(a)

$$\frac{m_3^2 - m_2^2}{m_3^2 + m_2^2} = \frac{\cos 2\theta_A}{1 - \sin^2 2\theta_A/2} + \mathcal{O}(|\epsilon|) \quad (9)$$

(b)

$$|V_{cb}| = |\operatorname{Re}\xi - i \cos 2\theta_A \operatorname{Im}\xi| + \mathcal{O}(|\epsilon^2|) \quad (10)$$

where  $\xi = \cos 2\theta_A (\epsilon_d - \epsilon_u)$  and  $\epsilon_i$  are the ratios between the relevant  $2^{nd}$  and  $3^{rd}$  generation masses.

(c)

$$\frac{m_\tau}{m_b} = 3 + 3 \sin 2\theta_A \operatorname{Re}[\epsilon_e - \epsilon_d] + \mathcal{O}(|\epsilon^2|) \quad (11)$$

at  $M_{GUT}$ . The prediction (a) is rather interesting, connecting the neutrino degeneracy with the largeness of the atmospheric mixing (which cannot obviously be maximal). The prediction (b) needs appreciable corrections in order to work. It connects nicely small quark and large lepton mixing, but ironically the quark mixing turns out too small.

Also, (c) as we commented before needs corrections (to be computed), since in the SM  $2|m_b| \approx |m_\tau|$ . A careful three generation numerical study is needed before one can know whether or not this theory works.

### 4.3 $10_H + 120_H$

In this case one must use  $16_H$  instead of  $126_H$  in order to break  $B-L$  and give  $\nu_R$  a mass. This allows for a radiative see-saw [38] at the two-loop level (Fig. 1), where one utilizes the gauge bosons in  $45_V$  and  $10_H$  in order to generate  $126_H$  effectively. This beautiful mechanism requires supersymmetry to be strongly broken due to the non-renormalization of the superpotential.

One can estimate

$$m_{\nu_R} \approx \left(\frac{\alpha}{\pi}\right)^2 Y_{10} \frac{M_R^2}{M_{GUT}} \frac{\tilde{m}}{M_{GUT}}, \quad (12)$$

where  $M_R$  is the  $SU(2)_R$  breaking scale, i.e.  $\langle 16_H \rangle$  and  $\tilde{m}$  the effective supersymmetry breaking scale in the visible sector (if  $\tilde{m} > M_{GUT}$ , then of course one would have  $M_{GUT}/\tilde{m}$  suppression).

In order to get  $m_{\nu_R}$  as large as possible,  $\tilde{m}$  should be as large as possible, which at first glance prefers no supersymmetry at all. However, then  $M_R \ll$

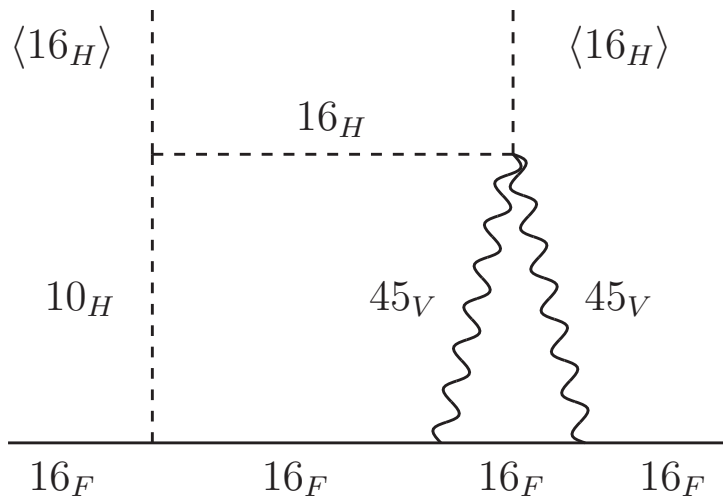


Figure 1: A contribution to the radiatively generated fermion mass.

$M_{GUT}$  in this case, since the single step unification does not work in the SM. Furthermore, one gets into trouble with  $b - \tau$  unification.

For a study of this model see [39, 40, 41], but under reasonable conditions one obtains the same predictions (a) and (b) as in (ii), with (c)  $m_b \approx m_\tau$  at  $M_{GUT}$ . The last prediction favors a split susy scenario with  $\tilde{m} \approx 10^9 - 10^{12}$  GeV (see Ref. [42] for a discussion of  $m_b \approx m_\tau$  in split susy).

Recently, the supersymmetric version of this Yukawa sector was studied in the context of charged fermions [43, 44]. Notice that the results in these papers do not apply to the theory we are discussing here, since in order to work in this case the situation favors strongly split supersymmetry. Actually, these works use the  $126_H$  for neutrinos, but decouple it from the charged fermions. This I find contrary to the main point of  $SO(10)$ , i.e. the connection between Dirac and Majorana Yukawas or the connection between charged fermions and neutrino masses. To me, it makes sense in the Pati-Salam theory where there is no connection between the two, but  $SO(10)$  is there in order to cure this.

## 5 Summary and outlook

I have tried in this short review to make a strong case for grand unification as a complete phenomenological theory (gravity ignored). The lack of observa-



tion of proton decay and magnetic monopoles made it for a long time basically impossible to define the minimal complete GUT. The hope is emerging now though through the information on fermion masses and mixings. Small non-vanishing neutrino masses point strongly towards SO(10), and SO(10) alone, without any new additional physics, offers predictive models. The minimal versions of the theory are good enough to be ruled out and the interesting model with low energy supersymmetry seems not to survive the fermion mass fitting and the unification constraints. Here though one must include the effects of supersymmetry breaking before claiming this model dead. The proper approach is to study this within the model which would provide a predictive model of soft terms. We plan to address this tedious project in near future.

What about giving up minimality? This was discussed off and on in the past, and recently again a case was made for a supersymmetric model with a full Yukawa sector in [43, 45, 46]. Not surprisingly these extended versions seems to pass the tests (they also seem to work in rather restricted portions of the parameter space); after all the minimal version almost made it and failed only when the tough constraints of the minimality of even the GUT Higgs sector were included. However, one should complete in my opinion the study of the other two equally minimal versions, the non-supersymmetric and the split supersymmetric ones, before giving up on the minimal theory. If finally one of the three possibilities I discussed survives, it will be important to compute all the proton decay rates.

It is interesting to note that the search for proton decay led to the discovery of atmospheric neutrino oscillations. Could it be that similarly on the theoretical side grand unification will turn out to be the theory of neutrino masses and mixings before being the theory of proton decay and magnetic monopoles? In any case, this program will succeed only if it manages to connect these phenomena in a predictive manner.

## 6 Added note

As I was preparing this manuscript for the net, two new papers appeared devoted to the in-depth study of the non-minimal model above, supplemented with the spontaneous CP violation [47]. This constraints the theory substantially and appears to be worth exploring.

Also, meanwhile a rather predictive theory emerged [48] based on a simple

extension of the minimal SU(5). It utilizes the so-called type III seesaw [49], and predicts light fermionic weak triplet with mass below TeV. This offers a remarkable possibility of seesaw possibly directly probed at LHC, since the decay of the triplet into charged leptons goes through its neutrino Yukawa couplings. Although this theory may lack the beauty and the depth of SO(10), its predictivity and the relevance for LHC makes a strong case in its favor.

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